

Dedicated to Holger Bech Nielsen on his 70th birthday.

Quantum Gravity in Plebanski Formulation

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Abstract

We present a theory of four-dimensional quantum gravity with massive gravitons which may be essentially renormalizable. In the Plebanski formulation of General Relativity (GR), in which the tetrads, the connection and the curvature are all independent variables (and the usual relations among these quantities are valid only on-shell), we consider a nonperturbative theory of gravity with a nonzero background connection. We predict a tiny value of the graviton mass: $m_g \approx 1.8 \times 10^{-42}$ GeV and an extremely small dimensionless coupling constant of the perturbative gravitational interaction: $g \sim 10^{-61}$. We put forward the idea by H. Isimori [56] on renormalizability of quantum gravity having multi-gravitons with masses m_0, m_1, \dots, m_{N-1} .

1 Introduction

Quantum gravity aims to solve the problem of merging quantum mechanics and general relativity, the two great conceptual revolutions in twentieth century physics. Gravity is the only fundamental interaction which cannot be considered as a full quantum theory. However, the theory of gravity is not complete yet. Currently, it cannot answer the following profound questions: What is the microstructure of spacetime explaining macroscopic gravitational interactions? What are the quantum origins of space, time and our Universe? Are "space", "time" and "causality" fundamental concepts? Where is the space-time geometry allowed to undergo large quantum fluctuations?

A lot of articles in literature are devoted to quantum gravity. We would like to mention the following books [1–16], reviews [17–19] and articles dealing with possible violations of Lorentz invariance at high energies [20–31]. The most important question in quantum gravity is whether the ground state of space-time obeys Lorentz invariance. The violation of Lorentz invariance appears in many approaches to quantum gravity. In addition, Lorentz-violating models also appear in noncommutative geometry [32, 33].

It is well-known that quantum gravity is a nonrenormalizable theory. However, in the present paper we investigate the four-dimensional quantum gravity which may be essentially renormalizable if one relaxes the assumption of metricity of the theory. Here we consider the Plebanski formulation [34] of general relativity in which the tetrads, the connection and the curvature are all independent dynamical variables, and the relations existing among them are valid only on-shell.

2 Plebanski theory of gravity

It has been showed [34–38] that the first-order formulation of gravity that uses tetrads instead of the metric is more fundamental. In the Plebanski formulation of theory of gravity [34] there arises a new independent field – the connection A^{IJ} , and the tetrad θ^I is used instead of the metric $g_{\mu\nu}$. Both A^{IJ} and θ^I are one-forms. Indices $I, J = 0, 1, 2, 3$ refer to the flat space-time with Minkowski metric η_{IJ} : $\eta^{IJ} = \text{diag}(-1, 1, 1, 1)$. This is a flat space which is tangential to the curved space with metric $g_{\mu\nu}$ at each its point. The world interval is represented as $ds^2 = \eta_{IJ}\theta^I \otimes \theta^J$, i.e. $g_{\mu\nu} = \eta_{IJ}\theta_\mu^I \otimes \theta_\nu^J$.

The action of the first-order gravity is [34–36]:

$$S(\theta, A) = \frac{1}{\kappa^2} \int_M \epsilon^{IJKL} \left(\theta^I \theta^J F^{KL} + \frac{\Lambda}{4} \theta^I \theta^J \theta^K \theta^L \right). \quad (1)$$

Here $\kappa^2 = 8\pi G$, where G is the gravitational coupling constant; $F = dA + (1/2)[A, A]$ is the curvature of the Lorentz group Lie algebra with spin connection A , and Λ is (a multiple of) the cosmological constant. The wedge product of all the forms is assumed.

The next step is to construct two-forms $\theta^I \wedge \theta^J$ and take their self-dual parts with respect to the indices I, J . Then we introduce an arbitrary time plus space split of the internal indices $I = (0, a)$, $a = 1, 2, 3$, and construct $F^a = \frac{1}{2}F_{\mu\nu}^a dx^\mu \wedge dx^\nu$ which is the field-strength two-form for gauge connection $A^a = A_\mu^a dx^\mu$. The field strength is written in component form as $F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + f^{abc} A_\mu^b A_\nu^c$, with $SO(3, C)$ structure constants $f^{abc} = \epsilon^{abc}$.

We try to get some new insight into the renormalization properties of 3+1 gravity by using the above action as a starting point. First, we see a problem that there is no quadratic term in (1) that could be interpreted as kinetic. Such a term arises only if one assumes the background to be constant (see below).

The main idea of Ref. [34] is that, in addition to the tetrad-like and connection fields, the theory contains a new field which on-shell becomes identified with the Weyl part of the curvature tensor. In this formulation gravity becomes a non-metric theory: instead of the tetrad one-forms one uses certain new two-forms that become related to the metric only on-shell.

In this paper we consider the original self-dual formulation of Plebanski [34–36]. In the units $\kappa = 1$, we adapt the starting action (1) to the language of the $\text{SO}(3, \mathbb{C})$ gauge algebra as

$$S(\Sigma, A, \psi) = \int_{M_4} (\Sigma^a \wedge F^a + (\Psi^{-1})_{ab} \Sigma^a \wedge \Sigma^b), \quad (2)$$

where $\Sigma^a = \frac{1}{2} \Sigma_{\mu\nu}^a dx^\mu \wedge dx^\nu$ is a triplet of the $\text{SO}(3, \mathbb{C})$ two-forms:

$$\Sigma^a = i\theta^0 \wedge \theta^a - \frac{1}{2} \epsilon^{abc} \theta^b \wedge \theta^c. \quad (3)$$

Here $a, b, c, \dots = 1, 2, 3$ are the $\text{SU}(2)$ Lie algebra indices. In Eq. (2) we have [35, 36]:

$$(\Psi^{-1})_{ab} = \Lambda \delta_{ab} + \psi_{ab}, \quad (4)$$

where ψ_{ab} is a field that on-shell becomes the Weyl part of the curvature, symmetric and traceless.

The equations of motion resulting from (2) are (see [36]):

$$\frac{\delta S}{\delta A^a} = D\Sigma^a = d\Sigma^a + \epsilon_{bc}^a A^b \wedge \Sigma^c = 0, \quad (5)$$

$$\frac{\delta S}{\delta \psi_{ab}} = \Sigma^a \wedge \Sigma^b - \frac{1}{3} \delta^{ab} \Sigma_c \wedge \Sigma^c = 0, \quad (6)$$

$$\frac{\delta S}{\delta \Sigma^a} = F^a - (\Psi^{-1})_b^a \Sigma^b = 0. \quad (7)$$

The Eq. (5) states that A^a is the self-dual part of the spin connection compatible with the two-forms Σ^a , where D is the exterior covariant derivative with respect to A^a . Eq. (6) implies that the two-forms Σ^a can be constructed from tetrad one-forms giving (3), which fixes the conformal class of the space-time metric $g_{\mu\nu} = \eta_{IJ} \theta_\mu^I \otimes \theta_\nu^J$ defined by the tetrads. The Eq. (7) states that the curvature F^a is self-dual as a two-form, which implies that the metric $g_{\mu\nu}$ derived from the tetrad one-forms θ^I satisfies the vacuum Einstein equations.

The 2-form fields Σ^a can therefore be integrated out of Eq. (2). Thus, we are led to Einstein's gravity given by the form:

$$S(A, \psi) = \int_M (\delta^{ab} \Lambda + \psi^{ab})^{-1} F^a \wedge F^b, \quad (8)$$

discussed in Refs. [35–38].

3 Nonperturbative theory of Plebanski gravity

Here we present a version of the nonperturbative gravity, assuming the condition $\Lambda \gg |\psi|$, i.e. considering ψ_{ab} as a perturbation to $\Lambda\delta_{ab}$, and use the following expansion:

$$(\delta^{ab}\Lambda + \psi^{ab})^{-1}F^a \wedge F^b \approx \frac{1}{\Lambda}(\delta_{ab} - \frac{1}{\Lambda}\psi_{ab} + \frac{1}{\Lambda^2}\psi_{ac}\psi_{cb} + \dots)F^a \wedge F^b. \quad (9)$$

According to Ref. [35], in the action (2) we have:

$$\Sigma^a \wedge \Sigma^a = -6i\sqrt{-g}d^4x, \quad (10)$$

what means that the dimensionless cosmological constant is equal to

$$\Lambda_0 = \frac{\rho_{vac}}{(M_{Pl}^{red.})^4} = 6\Lambda, \quad (11)$$

where ρ_{vac} is the effective vacuum energy density of the Universe and $M_{Pl}^{red.} = 1/\sqrt{8\pi G}$ is the reduced Planck mass.

Now we can calculate the partition function in Euclidean space:

$$Z = \int [\mathcal{D}A][\mathcal{D}\psi]e^{-S} \approx \int [\mathcal{D}A][\mathcal{D}\psi]\exp\left[-\frac{1}{\Lambda}\int_{M_4}(\delta_{ab} - \frac{1}{\Lambda}\psi_{ab} + \frac{1}{\Lambda^2}\psi_{ac}\psi_{cb} + \dots)F^a \wedge F^b\right]. \quad (12)$$

Taking into account that

$$F^a \wedge F^b = \frac{1}{4}F_{\mu\nu}^a F_{\rho\sigma}^b dx^\mu \wedge dx^\nu \wedge dx^\rho \wedge dx^\sigma = \frac{1}{4}F_{\mu\nu}^a F_{\rho\sigma}^b \epsilon^{\mu\nu\rho\sigma} \sqrt{g}d^4x, \quad (13)$$

we have:

$$F^a \wedge F^b = \frac{1}{2}F_{\mu\nu}^a F^{*b\mu\nu} \sqrt{g}d^4x, \quad (14)$$

where

$$F^{*b\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}^b \quad (15)$$

is a dual tensor. The requirement of self-duality $F^{*b\mu\nu} = F^{b\mu\nu}$ gives:

$$F^a \wedge F^b = \frac{1}{2}F_{\mu\nu}^a F^{b\mu\nu} \sqrt{g}d^4x. \quad (16)$$

Then the partition function is:

$$\begin{aligned} Z &= \int [\mathcal{D}A][\mathcal{D}\psi]e^{-S} \approx \int [\mathcal{D}A][\mathcal{D}\psi]\exp\left[-\frac{1}{2\Lambda}\int_{M_4}(\delta_{ab} - \frac{1}{\Lambda}\psi_{ab} + \frac{1}{\Lambda^2}\psi_{ac}\psi_{cb} + \dots)F^a \cdot F^b \sqrt{g}d^4x\right] \\ &= \int [\mathcal{D}A]\exp\left[-\frac{1}{2}\int_{M_4}\left(\frac{F^2}{\Lambda} + \ln\left(\frac{F^2}{2\Lambda M^4}\right)\right)\sqrt{g}d^4x\right], \end{aligned} \quad (17)$$

where $F^a \cdot F^b \equiv F_{\mu\nu}^a F^{b\mu\nu}$ and $F^2 = F_{\mu\nu}^a F^{a\mu\nu}$; M is the energy scale parameter.

Assuming the existence of the background B_μ^a of the connection A_μ^a :

$$A_\mu^a = B_\mu^a + \mathcal{A}_\mu^a, \quad (18)$$

where \mathcal{A}_μ^a is the perturbative connection, and $B^a \cdot B^a \equiv B_\mu^a B^{a\mu} = \text{const}$, we can calculate the partition function (17) and obtain the effective Lagrangian of the system:

$$Z = \int [\mathcal{D}\mathcal{A}] \exp \left(- \int L_{eff} \sqrt{g} d^4x \right). \quad (19)$$

Now we can return to the Minkowski spacetime. Using the gauge condition $\mathcal{A} \cdot B = 0$, it is easy to get the following result:

$$-L_{eff} = \frac{1}{2\Lambda} \left(\mathcal{F}^a \cdot \mathcal{F}^a + 2\sqrt{F_0^2} \mathcal{A}^a \cdot \mathcal{A}^a + F_0^2 \right) + \frac{1}{2} \ln \left(\frac{F_0^2}{2\Lambda M^4} \right), \quad (20)$$

where $\mathcal{F}^a \cdot \mathcal{F}^a \equiv \mathcal{F}_{\mu\nu}^a \mathcal{F}^{a\mu\nu}$, $\mathcal{F}_{\mu\nu}^a = \partial_\mu \mathcal{A}_\nu^a - \partial_\nu \mathcal{A}_\mu^a + f^{abc} \mathcal{A}_\mu^b \mathcal{A}_\nu^c$, and $\mathcal{A}^a \cdot \mathcal{A}^a \equiv \mathcal{A}_\mu^a \mathcal{A}^{a\mu}$.

In Eq. (20) we have:

$$F_0^2 = (B^a \cdot B^a)^2 = \text{const}, \quad (21)$$

which means that the vacuum expectation $\langle 0 | B^a \cdot B^a | 0 \rangle$ is nonzero (this condition is analogous to the gluon condensate in QCD). We neglected the term $(\mathcal{A}^a \cdot \mathcal{A}^a)^2$ in the Lagrangian (20), since \mathcal{A}_μ^a is a small field perturbation. In general, it is possible to take into account this quartic term as well.

Choosing $M^4 = \rho_{vac}$ and the condition:

$$F_0^2 = 2\Lambda \rho_{vac}, \quad (22)$$

we obtain the following physically reasonable result:

$$-L_{eff} = \frac{1}{4g^2} [\mathcal{F}^a \cdot \mathcal{F}^a + m^2 \mathcal{A}^a \cdot \mathcal{A}^a] + \rho_{vac}, \quad (23)$$

when the minimal effective potential density is equal to the vacuum density of the Universe: $\min U_{eff} = \rho_{vac}$.

In Eq. (23) the parameter m is related with a graviton mass:

$$m^2 = 2\sqrt{F_0^2} = 2\sqrt{2\Lambda \rho_{vac}} = \frac{2}{\sqrt{3}} \rho_{vac} (M_{Pl}^{red.})^{-2}. \quad (24)$$

Taking into account an estimate of cosmological measurements for the vacuum energy density ρ_{vac} of the Universe [39–44]:

$$\rho_{vac} \approx (2 \times 10^{-3} \text{ eV})^4, \quad (25)$$

and the well-known value $M_{Pl}^{red.} \approx 2.43 \times 10^{18} \text{ GeV}$, we can estimate the mass m :

$$m \approx \left(\frac{4}{3} \right)^{1/4} (2 \times 10^{-3} \text{ eV})^2 (2.43 \times 10^{18} \text{ GeV})^{-1} \approx 1.8 \times 10^{-42} \text{ GeV}. \quad (26)$$

This tiny value of the graviton mass m almost coincides with the present Hubble parameter value $H_0 \approx 1.5 \times 10^{-42} \text{ GeV}$ (see [39–44]).

Let us now estimate the effective coupling constant of the interaction of perturbative gravitational fields. From Eqs. (20)–(23) we see that the dimensionless coupling constant g is:

$$g^2 = \frac{\Lambda}{2}, \quad (27)$$

and according to Eq. (11), we have:

$$g = \sqrt{(\rho_{vac}/12)}(M_{Pl}^{red.})^{-2} \sim 10^{-61}. \quad (28)$$

We see that the perturbative gravitational interaction is extremely small. Nevertheless, the existence of a tiny graviton mass m_g leads to the renormalizability of the quantum gravity.

4 Massive graviton solution

Could a graviton be massive? The answer to this question seems to be positive [45–56]: if the graviton Compton wavelength, $\lambda_g = m_g^{-1}$, is large enough (\sim the present Hubble size H_0 , see the Eq. (26) of this paper), we should not be able to distinguish a massive graviton from a massless one. Astrophysical bounds are even milder [45–48].

A massive graviton in four-dimensions has five physical degrees of freedom (helicities $\pm 2, \pm 1, 0$) while the massless graviton has only two (helicities ± 2). In the limit $m_g \rightarrow 0$, the exchange by the three extra degrees of freedom can be interpreted as an additional contribution due to one massless vector particle with two degrees of freedom ("graviphoton" with helicities ± 1) and plus one real scalar particle ("graviscalar" with the helicity 0). The graviphotons do not contribute to the one-particle exchange: the contribution to the conserved energy-momentum tensor from graviphoton derivative coupling vanishes. The graviscalar is coupled to the trace of the energy-momentum tensor and its contribution is generically nonzero. This is what causes the difference between the theories of massless and massive gravitons.

The arguments of Refs. [49–55], based on the lowest tree-level approximation to the calculation of interactions between two gravitational sources, has a clear physical interpretation.

In the general case, a theory of massive gravitons possesses "ghosts". However, if the Lagrangian contains Pauli-Fierz mass terms [57], these "ghosts" are absent. But in the limit of vanishing graviton mass ($m_g \rightarrow 0$), the graviton propagator exhibits the Van Dam-Veltman-Zakharov (VDVZ) discontinuity [50–52] originating from the graviscalar which does not decouple in the massless limit $m_g \rightarrow 0$. At the classical level, the graviscalar doesn't cause problems [52–54]. But at the quantum level the theory becomes strongly coupled [58] (shows nonperturbative effects) at energy scale $(m^4 M_{Pl})^{1/5}$. This result was confirmed by explicit calculations in Ref. [59]. The phenomenon repeats in brane-world models when gravity is modified in the infrared limit [60–65].

Moreover, the nonlinear four-dimensional theory of the massive graviton is not defined unambiguously. For the vanishing graviton mass the lowest tree-level approximation breaks down, but the higher order corrections are singular in the graviton mass. The next-to-leading terms in the corresponding expansion are huge since they are inversely proportional to powers of m_g . Thus, the truncation of the perturbative series does not make much sense and all higher order terms in the solution of classical equations for the graviton field should be summed up. The summation leads to the nonperturbative solution which is continuous when $m_g \rightarrow 0$. The perturbative discontinuity shows up only at large distances where higher order terms are small. These distances are growing when $m_g \rightarrow 0$. In other words, the continuity is not perturbative and does not depend on distances considered in theory.

The reason for the problem of the lowest tree-level approximation is simple: it does not take into account the characteristic physical scale existing in theory. The nonperturbative calculation of the Schwarzschild solution leads to the elimination of this problem [52–54]. In the nonperturbative

approach, the coupling of the extra scalar mode to the matter is suppressed by the ratio of graviton mass to the characteristic physical scale which is Vainshtein's radius [52]: $R_V = (m_g^{-4} R_S)^{1/5}$, where R_S is the Schwarzschild's radius [53, 54]. The size R_V is large for small values of m_g . For example, if the graviton mass is given by Eq. (26), Vainshtein's radius is larger than Sun radius ($R_V \sim 100$ Kpc). Hence, the predictions of the massive theory could be made infinitely close to the predictions of the massless theory due to the tiny m_g .

It has been pointed out in Refs. [55, 66] that graviton mass terms violate Lorentz invariance. In Plebanski formulation the graviton field h^{ab} is described as a perturbation [35]:

$$\Sigma^a = \Sigma_0^a + \delta\Sigma^a = \Sigma_0^a + h^{ab}\bar{\Sigma}_0^b, \quad (29)$$

where the background two-forms are given by:

$$\Sigma_0^a = i dt \wedge dx^a - \frac{1}{2} \epsilon^{abc} dx^b \wedge dx^c, \quad (30)$$

and the anti-self-dual forms $\bar{\Sigma}^a$ are:

$$\bar{\Sigma}^a = i\theta^0 \wedge \theta^a + \frac{1}{2} \epsilon^{abc} \theta^b \wedge \theta^c. \quad (31)$$

Then

$$\bar{\Sigma}_0^a = i dt \wedge dx^a + \frac{1}{2} \epsilon^{abc} dx^b \wedge dx^c. \quad (32)$$

The first step is to find the linearized connection $\delta A^a = \mathcal{A}^a$ such that:

$$d\delta\Sigma^a + \epsilon^{abc} \mathcal{A}^b \wedge \Sigma^c = 0. \quad (33)$$

Then we obtain the equation of massive graviton:

$$\square h^{ab} - m_g^2 h^{ab} = 0, \quad (34)$$

where $m_g = m$ is given by the Eq. (26).

In the Minkowski space background, the metric $g_{\mu\nu}$ can be expanded as:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad (35)$$

where $\eta_{\mu\nu}$ is the flat metric of the Minkowski space. This expansion was first suggested by R.P. Feynman [1]. Then the equation of massive graviton is:

$$\square h^{\mu\nu} - m_g^2 h^{\mu\nu} = 0. \quad (36)$$

5 Multi-gravitons and renormalizable quantum gravity

To make the higher-derivative propagator, the author of Ref. [56] explores the model of multi-gravitons. He proposes the existence of ghost partners for gravitons.

Starting with the Einstein-Hilbert action:

$$L = \frac{1}{16\pi G} R + L_{mat}, \quad (37)$$

where L_{mat} is the matter Lagrangian, we have the Einstein equation:

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}, \quad (38)$$

where $T_{\mu\nu}$ is the energy-momentum tensor.

The gravitons are denoted by $h_{\mu\nu}^{(n)}$, where n runs from zero to $N-1$. The real gravitational field is:

$$h_{\mu\nu} = \sum_n h_{\mu\nu}^{(n)}. \quad (39)$$

All the gravitons are assumed to be massive with Pauli-Fierz mass terms [57]:

$$L_{mas} = -\frac{1}{32\pi G \sqrt{-g}} \sum_{n=0}^{N-1} (-1)^n m_n^2 (h_{\mu\nu}^{(n)} h^{(n)\mu\nu} - h^{(n)} h^{(n)}), \quad (40)$$

where $h = \eta_{\mu\nu} h^{\mu\nu}$, and:

$$\sqrt{-g} = 1 + \frac{1}{2}h - \frac{1}{4}h_\mu^\mu h_\nu^\nu + \frac{1}{8}hh + \dots \quad (41)$$

Considering L_{ghost} and $L_{kinetic}$, the author of Ref. [56]) obtains the following equation in the case $N=1$:

$$(\square - m_0^2)h_{\mu\nu}^{(0)} = -16\pi G \left(T_{\mu\nu} + \frac{1}{3} \left(\frac{\partial_\mu \partial_\nu}{m_0^2} - \eta_{\mu\nu} \right) T \right). \quad (42)$$

It is easy to write the equations of motion in a higher-derivative theory. For $N=2$ we have:

$$(\square - m_0^2)(\square - m_1^2)h_{\mu\nu} = (m_0^2 - m_1^2)16\pi G \left(T_{\mu\nu} + \frac{1}{3} \left(\frac{\partial_\mu \partial_\nu}{m_0^2} - \eta_{\mu\nu} \right) T \right). \quad (43)$$

Choosing the transverse-traceless gauge (TT-gauge) for all gravitational fields, we can quantize gravitational wave of each graviton field, which was performed in Ref. [56].

Let us calculate propagators for small N . They help to calculate the vacuum energy and graviton mass corrections. According to the Feynman rules, the propagator for $N=2$ is:

$$D_{\mu\nu\rho\sigma} = -i \frac{(m_1^2 - m_0^2)}{(p^2 + m_0^2)(p^2 + m_1^2)} P_{\mu\nu\rho\sigma}, \quad (44)$$

where

$$P_{\mu\nu\rho\sigma} = \frac{1}{2}(\eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho} - \frac{2}{3}\eta_{\mu\nu}\eta_{\rho\sigma}). \quad (45)$$

In this case the theory of gravity is renormalizable: the number of counter terms are finite.

If we wish to avoid the finetuning, then the case $N=2$ is not enough. To make a super-renormalizable model [56], the number of counter terms are reduced, we must use $N=4$ with the assumption: $m_3 = \sqrt{m_0^2 - m_1^2 + m_2^2}$. Then

$$D_{\mu\nu\rho\sigma} = -i \frac{(m_1^2 - m_0^2)(m_1^2 - m_2^2)(m_0^2 + m_2^2 + 2p^2)}{(p^2 + m_0^2)(p^2 + m_1^2)(p^2 + m_2^2)(p^2 + m_3^2)} P_{\mu\nu\rho\sigma}. \quad (46)$$

In the same way, we can get a super-renormalizable model for any even N .

Theory [56] preserves unitarity. The higher-derivative propagators suppress divergences of the vacuum energy and graviton mass corrections. Applying ghost partners for the Standard Model particles, quantum gravity with matter fields becomes renormalizable with power counting arguments.

6 Conclusions

In the present investigation we have used the formulation of general relativity (GR) in terms of self-dual two-forms due to Plebanski [34]. Using the formulations given in Refs. [34–36], we demonstrated that the Plebanski theory presents quite an economical alternative to the usual metric and frame-based schemes of GR. In Section 3 we have developed the nonperturbative theory of gravity with a nonzero background connection B_μ^a : $A_\mu^a = B_\mu^a + \mathcal{A}_\mu^a$, where \mathcal{A}_μ^a is a small perturbative connection and $a = 1, 2, 3$ is the $SU(2)$ Lie algebra index. We have assumed that $\langle 0|B_\mu^a B^{a\mu}|0 \rangle = \text{const.}$ This condition is analogous to the gluon condensate in QCD. Our prediction gives a tiny value of the graviton mass: $m_g \approx 1.8 \times 10^{-42} \text{ GeV}$ which almost coincides with the present Hubble parameter value H_0 [39–44]. In this case, we should not yet be able to distinguish a massive graviton from a massless one [52–54]. We also predicted an extremely small dimensionless coupling constant of the interaction of perturbative gravitational fields: $g \sim 10^{-61}$.

Considering the problem of renormalizability of quantum gravity, we briefly reviewed the model of multi-gravitons by H. Isimori [56] with N gravitons having masses m_0, m_1, \dots, m_{N-1} . For $N = 2$, gravity is renormalizable and the number of counter terms is finite. If $N = 4$, then the quantum theory of gravity is super-renormalizable (the number of counter terms is reduced), and such a theory avoids any finetuning problems.

We hope to develop the quantum theory of multi-gravitons in our forthcoming communications.

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